Friday, 10:15 - 12:15

■ FC-03

Friday, 10:15 - 12:15
23.3.4

Optimal Control 1

Contributed session

Chair: *Peter Roebeling*, Department of Environment and Planning, CESAM - University of Aveiro, Campus Universitário de Santiago, Universidade de Aveiro, 3810-193, Aveiro, Portugal, peter.roebeling@ua.pt

1 - Necessary optimality conditions for optimal control problems with discontinuous right hand side

Werner Schmidt, Ernst-Moritz-Arndt-University Greifswald, D-17489, Greifswald, Germany, wschmidt@uni-greifswald.de, *Olga Kostyukova*, *Ekaterina Kostina*

We consider an optimal control problem with discontinuous right-hand side. It is assumed that the system dynamics is switched when system crosses a given surface described by a smooth function dependent on system states. Attention is paid to the situation when optimal trajectory slides on switching surface during nontrivial intervals. Necessary optimality conditions in a form the Maximum Principle are proved. The principle includes new essential conditions. Comparison of the optimality conditions obtained with some other known results is carried out. Illustrative examples are presented.

2 - Invexity in mathematical programming and control problems

Manuel Arana-Jiménez, Estadistica e Invesitigacion Operativa, University of Cadiz, C/Chile, 1, 11002, Jerez de la Frontera, Cadiz, Spain, manuel.arana@uca.es, *Antonio Rufián-Lizana*, *Gabriel Ruiz-Garzón*, *Rafaela Osuna-Gómez*

This communication is focused on the study of optimal solutions for mathematical programming problems and control problems and the properties of the functions, as well as the relationship between these types of optimization problems, from recent published results. KT-invexity has been introduced in control problems, and it is a necessary and suffcient condition in order for a Kuhn-Tucker critical point to be an optimal solution. Recenty, a weaker condition, FJ-invexity, has been proposed, which is characterized by a Fritz John point being an optimal solution for the control problem.

3 - Efficient inter-industry water pollution abatement in linked terrestrial and marine ecosystems

Peter Roebeling, Department of Environment and Planning, CESAM - University of Aveiro, Campus Universitário de Santiago, Universidade de Aveiro, 3810-193, Aveiro, Portugal, peter.roebeling@ua.pt, *Eligius M.T. Hendrix*, *Arjan Ruijs*, *Martijn van Grieken*

Catchment agriculture leads to water pollution, downstream environmental degradation and marine value depreciation. Sustainable development entails balancing of marginal costs and benefits from pollution abatement. As abatement costs differ across agricultural industries and abatement benefits are nonlinear, we explore efficient abatement across industries. Using an optimal control approach with application to cane and cattle industries in Tropical Australia, we show that efficient abatement per industry is dependent on abatement by all industries when marine abatement benefits are non-linear.

■ FC-04

Friday, 10:15 - 12:15
23.3.5

Generalized differentiation and applications

Invited session

Chair: *Vera Roshchina*, CIMA, Universidade de Evora, Colégio Luís Verney, Rua Romão Ramalho, 59, 7000-671, Évora, Portugal, vera.roshchina@gmail.com

1 - Minimizing irregular convex functions: Ulam stability for approximate minima

Michel Théra, Maths-Info, XLIM, UMR-CNRS 6172, 123, Avenue Albert Thomas, 87060, Limoges Cedex, France, michel.thera@unilim.fr

This presentation summarizes a recent joint work with Emil Ernst. Our main objective is to characterize the subclass of those convex lower semicontinuous proper functions bounded below for which the set-valued mapping which assigns to a function in this class, the set of its epsilon-minima is upper semi-continuous. Despite its abstract appearance, this type of stability turns out to be essential in numerical optimization, namely in answering the natural question of defining the largest class of functionals convex lower semicontinuous proper and bounded below for which minimization algorithms exist.

2 - The Directed Subdifferential

Elza Farkhi, School of Math. Sciences, Tel-Aviv University, Haim Levanon Str., 69978, Tel Aviv, elza@post.tau.ac.il, *Robert Baier*

For differences of convex functions the directed subdifferential is introduced as the difference of two embedded convex subdifferentials in the Banach space of directed sets. Basic axioms of subdifferentials and nice calculus rules are established for the directed subdifferential. Its visualization, called Rubinov subdifferential, is a non-empty, generally non-convex set in Rn. Optimality conditions are formulated, minimizers, maximizers and saddle points are distinguished, directions of descent and ascent are identified using the directed and Rubinov subdifferential.

3 - Calculating Known Subdifferentials from the Rubinov Subdifferential

Robert Baier, Department of Mathematics, University of Bayreuth, Chair of Applied Mathematics, D-95440, Bayreuth, Germany, robert.baier@uni-bayreuth.de, *Elza Farkhi*, *Vera Roshchina*

The visualization of the directed subdifferential - defined for differences of convex functions - is the Rubinov subdifferential. This set is usually non-convex and splits into three parts. The relation between these parts and the Dini, Michel-Penot and Clarke subdifferential are discussed. In 2D the Rubinov subdifferential is closely linked to the Mordukhovich one and offers the calculation of various subdifferentials based on simple differences of sets. Several visualizations in examples indicate the connections to other subdifferentials and the advantage of nonconvex subdifferentials.

4 - Subgradient sampling algorithms for nonsmooth nonconvex functions

Adil Bagirov, School of Information Technology & Mathematical Sciences, University of Ballarat, University Drive, Mount Helen, P.O. Box 663, 3353, Ballarat, Victoria, Australia, a.bagirov@ballarat.edu.au

In this talk we will present an algorithm for computation of subgradients of nonsmooth nonconvex functions. Then we demonstrate how this algorithm can be applied to approximate subdifferentials and quasidifferential. We also discuss an algorithm for the computation of descent directions of such functions. Examples wiil be given to demonstrate the performance of the algorithms.